Incorporating Memory Into Financial Market Models

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Introduction

Standard financial/economic models assume:

- Rational agents with unlimited processing power (Homo Economicus)
- Maximized utility functions/competitive equilibrium
- Correct and uncorrelated information
- Rational expectations — The ‘Representative Agent’
- Perfect liquidity,...

Result:
Current prices accurately incorporate all past information and the system is Markovian.

The reality:

- Irrational/psychological/memory effects
- Perverse incentives/moral hazard
- A 'less-than-perfect' information stream
- Significant coupling between agents eg. common (mis)conceptions, herding
- Liquidity risk, counterparty risk,...

Result ??:
<u>The stylized facts</u>

- fat-tails — non-Gaussian decay of price returns
- volatility clustering — heteroskedasticity
- Excess volatility and trading

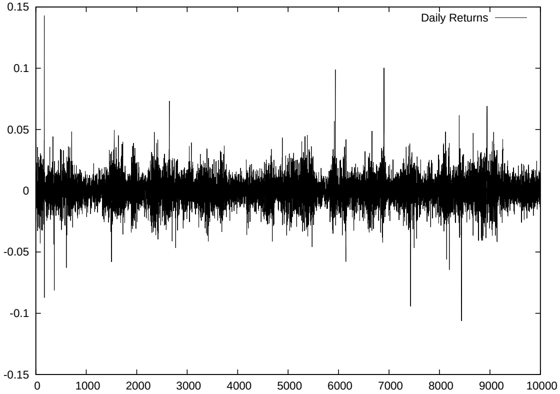
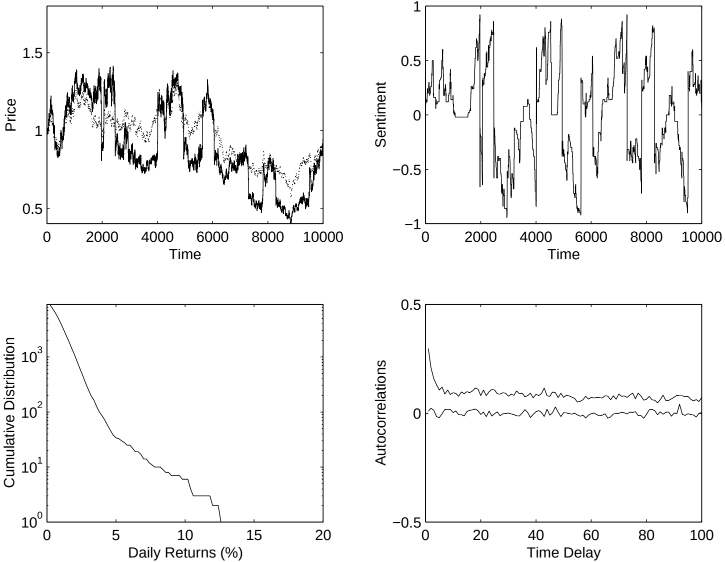


Figure: Simulation over 40 years (10000 time intervals)



Price
1.5
1
0.5
0
2000
4000
6000
8000
10000
Time
Sentiment
1
0.5
0
−0.5
−1
0
2000
4000
6000
8000
10000
Time
Cumulative Distribution
10^3
10^2
10^1
10^0
0
5
10
15
20
Daily Returns (%)
Autocorrelations
0.5
0
−0.5
0
20
40
60
80
100
Time Delay

Arguments against Markovian markets

Micro-level:

- Anchoring
- Agents may have preferences (with framing) rather than utility functions
- Bounded rationality
- The concept of profit/loss is inherently non-Markovian
- Hysteresis

In addition, at the Macro-level:

- Economic shocks can have long-lasting effects...
- ... as can policy decisions

Remark: Volatility clustering is not necesarily a sign of market memory.

Alternatives to the Markovian assumption

- The 'unit-root' explanation
- Memory-free but with ‘hidden variables’ and sensitivity to initial conditions
- Hysteresis
- True path-dependence

Price-generating Models

These are attempts to mimic volatility clustering by replacing Brownian Motion with

- GARCH
- FIGARCH
- Fractional Brownian Motion
- ...

No economic understanding is required or generated.

Probabilistic Switching Models

Lux-Marchesi (1999).

There are M agents — either ‘Fundamentalists’ (memory-free) or ‘Chartists’.

Agents can switch between the two groups.

There is a critical proportion of Chartists that causes bubbles etc

The effects disappear as M increases.

Probabilistic switching is unrealistic and leaves no room for behavioural eonomics.

Artificial Markets

The Santa-Fe Institute

Agents choose the best-performing strategies at their disposal (à la El Farol).

Memory is present via evolutionary effects on both beliefs and prices.

The decision-making processes are very complicated.

Hysteresis

Simple *hysterons* are often used to mimic sunk-costs.

The aggregate behaviour depends only upon a subset of past input extrema.

However the concept of a unique equilibrium is gone.

A Threshold-based Model

Cross, Grinfeld, L. & Seaman.

The usual, efficient market, pricing formula (drift-free geometric Brownian motion) is:

$$p_{n+1} = p_n \exp\left[\sqrt{h}\Delta W_n - \frac{h}{2}\right].$$

We replace this by:

$$p_{n+1} = p_n \exp\left[\left(\sqrt{h}\Delta W_n - \frac{h}{2}\right) f(\sigma_n) + \kappa\Delta\sigma_n\right].$$

- Agent i switches when $p(n) \notin [L_i(n), U_i(n)]$.
- After each switching a new $[L_i, U_i]$ is generated.

Modeling Assumptions

We introduce a separation of timescales:
'Fast agents' — react to exogenous information/noise.
'Slow agents' — react to changes in price.

The 'Price Threshold Assumption':

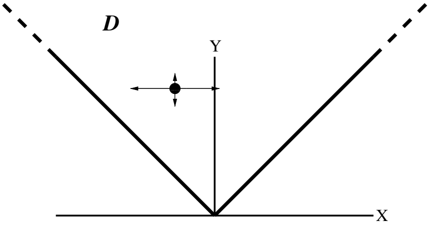
These threshold values contain all of the past experience and future expectations of the slow agents.

The fast agents are not simulated directly.

Remarks

- The market is non-Markovian
- Each pair of thresholds is a current ‘trading strategy’
- The threshold dynamics can mimic important phenomena — behavioural economics, inductive learning, ...
- The anchoring effect and transaction costs/inertia are automatically captured
- If $f \equiv 1,\ \sigma_0 \approx 0$ **AND there is no coupling between agents** then we obtain efficient market pricing.
- System properties are independent of M.

Herding: For agents *in the minority only* the thresholds move together.



Perverse incentives can be modelled in a similar manner.

Volatility clustering can be mimicked by setting $f(\sigma_n) = 1 + \alpha|\sigma_n|$.

Remember that herding may be irrational or rational-but-perverse.

Closing Questions

- How are assumptions unwound — all together or LIFO? Do different agents unwind at different speeds?
- How to model the reinterpretation of old information? Is this the cause of volatility clustering?
- Can all memory effects be adequately modelled using price history?
- Analagous to rational expectations, which memory effects cancel and which do not?